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Modelling of an ilmenite-smelting DC arc furnace process

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Abstract

Ilmenite smelters must be operated with the slag bath contained in a freeze lining of solidified slag to prevent refractory damage and ensure feasible furnace life. This makes the process an interesting and challenging one to operate and control.

The study focused on interactions between the freeze lining and slag bath. A mathematical model was developed to describe heat transfer, solidification and melting in the freeze lining and furnace wall. This model was incorporated into another model to yield a dynamic simulation-type model of the entire smelting furnace process. Simulation experiments with the models aimed to study the dynamic response of the freeze lining to changes in power input, slag composition, and operating parameters.

Due to confidentiality considerations, it was not possible to incorporate industrial data for comparison into the study. Regardless of this, the results still remain useful to practitioners when considering the process aspects that were investigated.

Keywords: Pyrometallurgy; Modelling; Simulation

1. Introduction

Large pyrometallurgical furnace processes often contain much complexity and uncertainty that make it difficult to reason intuitively about their dynamic behaviour. Ilmenite-smelting furnaces in particular must be operated with a freeze lining to prevent attack of the sidewall refractory material by corrosive liquid high-titania slag. Because the freeze lining consists of slag solidified from the slag bath, it interacts both chemically and thermally with the slag bath. This interaction represents a delicate balance in the process that must be maintained by carefully controlling the relative amounts of the process inputs (ilmenite, reductant and electrical energy). The result is that an ilmenite-smelting furnace process presents a number of interesting process control challenges. More details about the ilmenite-smelting DC arc furnaces process are provided elsewhere (Zietsman, 2004).

This study (Zietsman, 2004) focused on interactions between the slag bath and freeze lining. Specifically of interest were the dynamic interactions and dynamic behaviour of the process resulting from influences exerted on the system by the slag bath thermal and chemical state, and by operational parameters such as ilmenite feed rate, reductant feed rate and power input.

The motivation for undertaking the study (Zietsman, 2004) was to improve the understanding of the freeze lining behaviour because of the importance of the freeze lining to a number of facets of an ilmenite-smelting operation. The models and new knowledge could then be applied to investigate the impact of positioning of thermocouples in the furnace sidewall, to investigate and experiment with process control strategies that would improve the performance actual furnaces, and to study the underlying process mechanisms active in the process (Zietsman and Pistorius, 2004).

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In this study, a simplified slag system consisting of only FeO, TiO_2 and Ti_2O_3 was considered. The work described in this paper was part of a Ph.D. study at the University of Pretoria (Zietsman, 2004).

2. Approach of the study

The approach of this study only involved mathematical modelling. Due to the secrecy surrounding all ilmenite-smelting operations, no industrial data could be used in the study for validating the models.

The steps in the study were as follows:

- Create a model (Wall Model) of the furnace wall, freeze lining and slag bath in isolation without considering other process components such as the metal bath, freeboard etc.
- Do a focused investigation of the interactions between the slag bath and freeze lining with the Wall Model.
- Create a model (Crust Model) to describe crust formation on top of the slag bath.
- Create a model (Furnace Model) to describe the complete ilmenite-smelting furnace process and build into it both the Wall Model and Crust Model.
- Do an investigation into the influence of changes in operational parameters on slag bath and freeze lining.
- Review the mechanisms active in the process (Zietsman and Pistorius, 2004).

All of the above aspects, except for the review on process mechanisms, are reported on in this paper.

3. The Wall Model

3.1. Purpose

The purpose of this model was to describe the dynamic response of the furnace sidewall and freeze lining resulting from interactions with the liquid slag bath and changing conditions in the bath.

3.2. Modelled system

The system described by the Wall Model is shown in Fig. 1. The layers that were included in the model were the slag bath, freeze lining, refractory brick layer, ramming material layer and the steel shell (inner shell).

3.3. Key phenomena

The key phenomena addressed in the Wall Model are summarised in Table 1. It was attempted to identify all relevant phenomena in the system as depicted in Fig. 1 and then assign a level of importance to each. A level of importance of 1 means that the phenomenon was considered unimportant and it was subsequently ignored. Level 2 indicates that the phenomenon had to be incorporated into the model, but it was not necessary to model it in detail. Level 3 marks critical phenomena that had to be modelled in as much detail as possible.

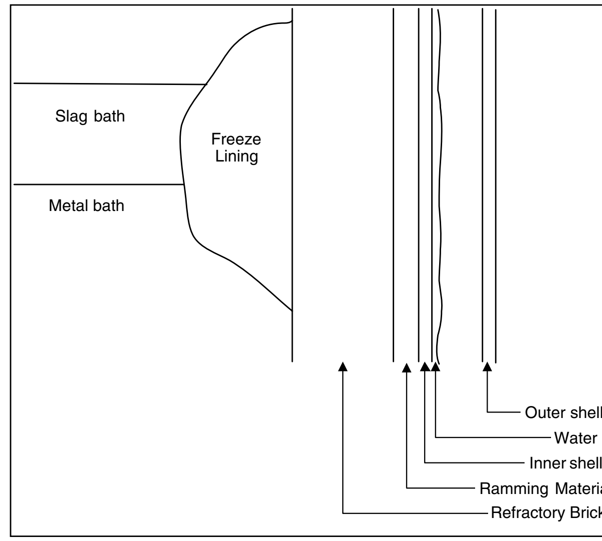


Fig. 1. Schematic representation of the freeze lining and wall region of an ilmenite-smelting DC arc furnace.

3.4. Formulation

The detailed model formulation was published elsewhere (Zietsman, 2004). Only the most important aspects are highlighted here.

3.4.1. Assumptions

The following assumptions were made as part of the formulation of the Wall Model:

- Axial and angular heat transfer components are negligible and can be ignored.

  The *radial dimension* therefore became the 'focus' dimension in the model. Radial heat transfer refers to transfer of heat in the direction from the centre of the circular furnace towards the furnace wall in a horizontal plane and in the direction opposite to this from the outside towards the centre of the circular furnace in a horizontal plane.

  Two modes of heat transfer in the radial dimension were considered, namely conduction and convection. *Conduction* refers to transfer of heat in and between the solid layers of the furnace wall and freeze lining. *Convection* refers to transfer of heat from the liquid slag bath to the solid freeze lining and vice versa. Convection heat transfer was handled in a simplified way by using an effective thermal conductivity approach (Brimacombe, 1976; Zietsman, 2004).

Table 1
Summary of key phenomena for the Wall Model

			Level of importance		
			1	2	3
Heat transfer	Radial	In slag bath		✓	
		Slag bath to freeze lining			✓
		In freeze lining			✓
		Freeze lining to refractory brick			✓
		In refractory brick			✓
		Refractory brick to ramming material			✓
		In ramming material			✓
		Ramming material to steel shell			✓
		In steel shell			✓
		Steel shell to water		✓	
	Axial	In slag bath	✓		
		In freeze lining	✓		
		In refractory brick	✓		
		In ramming material	✓		
		In steel shell	✓		
		In water	✓		
	Angular	In slag bath	✓		
		In freeze lining	✓		
		In refractory brick	✓		
		In ramming material	✓		
		In steel shell	✓		
		In water	✓		
	Heat sources	Electric arc			✓
		Slag solidification			✓
		Chemical reaction			✓
	Heat sinks	Slag melting			✓
		Chemical reaction			✓
		Water cooling		✓	
Mass transfer	Convection	In slag bath	✓		
Momentum transfer	Diffusion	In slag bath	✓		
		In freeze lining	✓		
	Slag bath			✓	
	Metal bath		✓		
	Water cooling layer		✓		
Chemical reaction	Liquid slag	With liquid slag	✓		
	Liquid slag	With solid slag			✓
	Liquid slag	With refractory material	✓		
	Solid slag	With solid slag	✓		
	Solid slag	With refractory material	✓		
Mechanical effects	Tapping equipment		✓		
	Buoyancy in metal bath		✓		

- The slag bath is well mixed and can therefore be treated as ideally mixed.
- Mass transfer and solid-state chemical reaction in the freeze lining are negligible and can be ignored.
- There is a strong drive towards thermodynamic equilibrium at the interface between the slag bath and freeze lining. An equilibrium calculation can therefore be used to describe solidification and melting at the interface.
- The temperature dependence of the density of materials (liquid slag, solid slag, magnesia brick, ramming material and steel) used in this model is negligible and can be ignored. Convection heat transfer in the slag was described by an effective thermal conductivity approach (Brimacombe, 1976; Zietsman, 2004).
- The thermal conductivity of solid slag is assumed to be 0.001 kW/(m °C), and independent of temperature (Pistorius, 2004).

- No liquid metal reaches the freeze lining. Liquid metal therefore does not have to be included as an input to the interface between the slag bath and freeze lining.As shown in Fig. 1 this is not the case in the actual furnace. However, to simplify the system to be modelled, an axial level in the furnace was chosen where the freeze lining never comes into contact with liquid slag. This is the reason for depicting the system as shown in Fig. 2.
- The outer surface of the steel shell is at a constant temperature. This simplifies heat transfer calculations at that boundary.
- The contact resistances between the layers (solid slag, brick, ramming, steel) of the conductor are not rate-determining of heat transfer through the wall. These resistances could therefore be made as large as numerical stability of the model would allow.

3.4.2. Simplifications

To simplify the model, it was decided that the influence of contact between the metal bath and the freeze lining on freeze lining behaviour would be ignored.

3.4.3. Model structure

The model was constructed using a combination of the one-dimensional finite difference method (Holman, 1989) to describe heat transfer through each layer, and free energy minimisation (Eriksson and Hack, 1990) to calculate the local equilibrium at the interface between the slag and metal baths. The radial dimension was modelled and the axial and angular dimensions ignored. Fig. 2 shows the structure.

The heat transfer part of the model described heat transferred by convection from the slag bath to the freeze lining, and by conduction through the freeze lining, refractory brick, ramming and steel layers. An 'effective' convection heat transfer coefficient was used for heat transfer across the contact between the layers. On the outside of the steel layer a constant temperature assumption was used, which resulted in the heat transfer rate at this interface varying in response to changing conditions.

3.4.4. Boundary conditions

The two most important boundary conditions in this model are a constant temperature (50 °C) boundary condition on the outside of the steel shell of the furnace wall, and an effective thermal conductivity boundary condition for heat transfer between the liquid slag bath and the solid freeze lining. The value of the effective thermal conductivity that was used for the liquid slag was 0.005 kW/(m °C).

3.5. Solution

To apply the Wall Model, it was incorporated into the modelling framework proposed by Pauw (1989) as a conductor module (see Appendix A—-Model element descriptions). A flowsheet (Fig. 3) was constructed within which the Wall Model could be tested and applied during investigations.

The flow sheet uses the Slag Feed to initially fill the Slag Bath with liquid slag. The Slag Bath is used to contain all liquid slag in the system. The Furnace Wall conductor module contains all solid slag, refractory brick, ramming material and the steel shell. Liquid slag is circulated from the Slag Bath to the Furnace Wall and back to represent the circulation of liquid slag in an actual furnace. The Metal Bath receives all liquid metal resulting from precipitation during slag solidification at the Furnace Wall. The Electrical Power module is used to supply energy to the system and the Heat Losses module to remove energy from the system.

The entire flow sheet is a graphical representation of a set of differential equations. These equations are integrated over a series of time steps to provide model outputs. For more detail on, and for a mathematical presentation of this modelling technique and the solution method, the reader is referred to a text dedicated to this subject (Pauw, 1989).

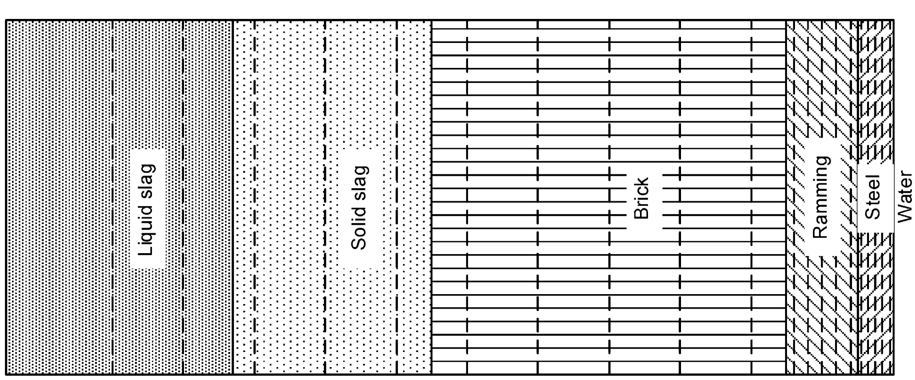


Fig. 2. Schematic representation of the structure of the Wall Model.

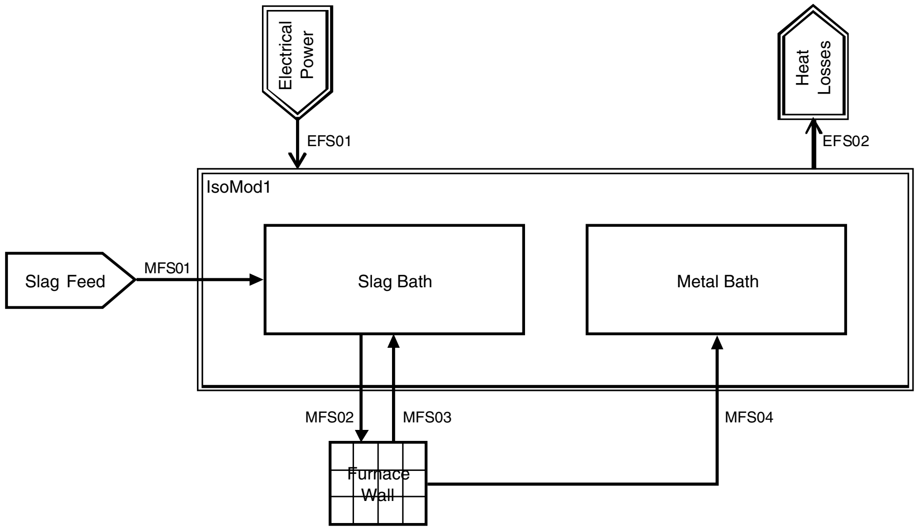


Fig. 3. Flow sheet of a simple process model incorporating the Wall Model.

3.6. Validation

The Wall Model was rigorously tested against known analytical solutions of the same problem. Good agreement was found between the analytical and numerical solution result sets. The model could not be tested with industrial data due to the confidentiality of such data.

3.7. Limitations

The greatest limitation of the current Wall Model is the fact that it describes the system in only one dimension, the radial dimension. Contact between the liquid metal bath and the lower refractory walls of the furnace and heat transfer across this interface are likely to induce significant axial heat transfer components in the lower sidewalls. The current Wall Model is not able to describe this. A two-dimensional version of the Wall Model is currently being developed to eliminate this limitation.

4. Influence of input heat flow rate on freeze lining and slag bath interactions

4.1. Objectives

In the first set of simulation experiments conducted with the Wall Model, it was aimed to study thermal influences on the freeze lining and its interaction with the slag bath in isolation. This is, of course, not completely possible due to the thermal and chemical behaviour of the system being dependent on one another. The following aspects were investigated and will be reported on here:

- Freeze lining thickness.
- Temperature distribution through the furnace wall and freeze lining.
- Liquid slag temperature.
- Composition distribution through the freeze lining.
- Liquid slag composition.

4.2. Experimental setup

The flow sheet shown in Fig. 3 was used to conduct the study. The Slag Bath was filled with a slag of specified composition. The Furnace Wall was initialised with a freeze lining of set thickness and composition, and with a specific steady state temperature profile through all the layers of the conductor module. The Metal Bath started off empty.

The dimensions and capacities of the system are listed in Table 2. The physical properties of materials used in this study are included in Table 3. All experiments were run for a simulated time of 24 h, or until the freeze lining disappeared.

4.3. Experiments conducted

The parameters that were varied over the series of simulation experiments include the following:

- The initial steady state heat flow rate, which implies a specific freeze lining thickness.
- Initial freeze lining composition.
- Net input heat flow rate. This was manipulated by setting the energy flow rate of the Electrical Energy and Heat Losses modules on the flow sheet.

The list of experiments is shown in Table 4.

The set of experiments contained three subsets. Set I forms the base with an initial steady state heat flux of 250 kW and the initial freeze lining consisting of only

Table 2
Dimensions and quantities of the system used in Wall Model experiments

Description	Dimension/quantity
Steel layer outer radius	5.000 m
Ramming layer outer radius	4.975 m
Brick layer outer radius	4.925 m
Brick layer inner radius	4.425 m
Allowed freeze lining thickness	0.000–2.000 m
System height	1.000 m
Total slag mass in system	±421 ton
System height	1.000 m

pseudobrookite. In set II the initial steady state heat flux was increased to 300 kW and in set III the initial freeze lining was changed to Rutile.

4.4. Detailed results

An example of a detailed result set of one of the experiments (6.1) is shown in Fig. 4. The same result set format was also used while studying the influence of slag bath composition. This is described in the next section.

Table 3
Material properties used in this study

Material	Composition	Density [kg m^{-3}]	Thermal conductivity [kW m^{-1} °C^{-1}]
Steel (Perry and Green, 1997)	1.2% C, 0.3% Mn	7800	$2.0792 \times 10^{-8} \times T^2 - 4.69295 \times 10^{-5} \times T + 5.6235 \times 10^{-2}$
Graphite ramming material (Weast and Astle, 1983)	100% C (solid graphite)	1700	$1.066 \times 10^{-1} \times e^{-0.0011805 \times T}$
Magnesia brick (Ruh and McDowell, 1962)	93.6% MgO	2787	$1.1511 \times 10^{-2} \times e^{-0.0011304 \times T}$
Solid slag (Pistorius, 2004)	Values assumed to be valid for various typical high-TiO_2 slag compositions	3800	0.001
Liquid slag (Pistorius, 2004)	Values assumed to be valid for various typical high-TiO_2 slag compositions	3800	0.001
Liquid metal	Dependant on the circumstances modelled	Not used	Not used

Table 4
Experiments conducted to study thermal influences with the Wall Model

Subset	Experiment no.	Initial steady state heat flow (kW)	Initial freeze lining thickness (m)	Initial freeze lining composition	Electrical power heat flow (kW)	Heat losses heat flow (kW)	Net input heat flow (kW)
I	6.1	250	0.102	Pseudobrookite	0	1000	−1000
I	6.2	250	0.102	Pseudobrookite	0	500	−500
I	6.3	250	0.102	Pseudobrookite	0	200	−200
I	6.4	250	0.102	Pseudobrookite	0	100	−100
I	6.5	250	0.102	Pseudobrookite	0	0	0
I	6.6	250	0.102	Pseudobrookite	100	0	100
I	6.7	250	0.102	Pseudobrookite	200	0	200
I	6.8	250	0.102	Pseudobrookite	300	0	300
I	6.9	250	0.102	Pseudobrookite	400	0	400
I	6.10	250	0.102	Pseudobrookite	500	0	500
I	6.11	250	0.102	Pseudobrookite	1000	0	1000
I	6.12	250	0.102	Pseudobrookite	2000	0	2000
I	6.13	250	0.102	Pseudobrookite	5000	0	5000
I	6.14	250	0.102	Pseudobrookite	10,000	0	10,000
II	6.15	300	0.066	Pseudobrookite	0	1000	−1000
II	6.16	300	0.066	Pseudobrookite	0	0	0
II	6.17	300	0.066	Pseudobrookite	200	0	200
II	6.18	300	0.066	Pseudobrookite	400	0	400
II	6.19	300	0.066	Pseudobrookite	1000	0	1000
II	6.20	300	0.066	Pseudobrookite	10,000	0	10,000
III	6.21	250	0.102	Rutile	0	1000	−1000
III	6.22	250	0.102	Rutile	0	0	0
III	6.23	250	0.102	Rutile	200	0	200
III	6.24	250	0.102	Rutile	400	0	400
III	6.25	250	0.102	Rutile	1000	0	1000
III	6.26	250	0.102	Rutile	10,000	0	10,000

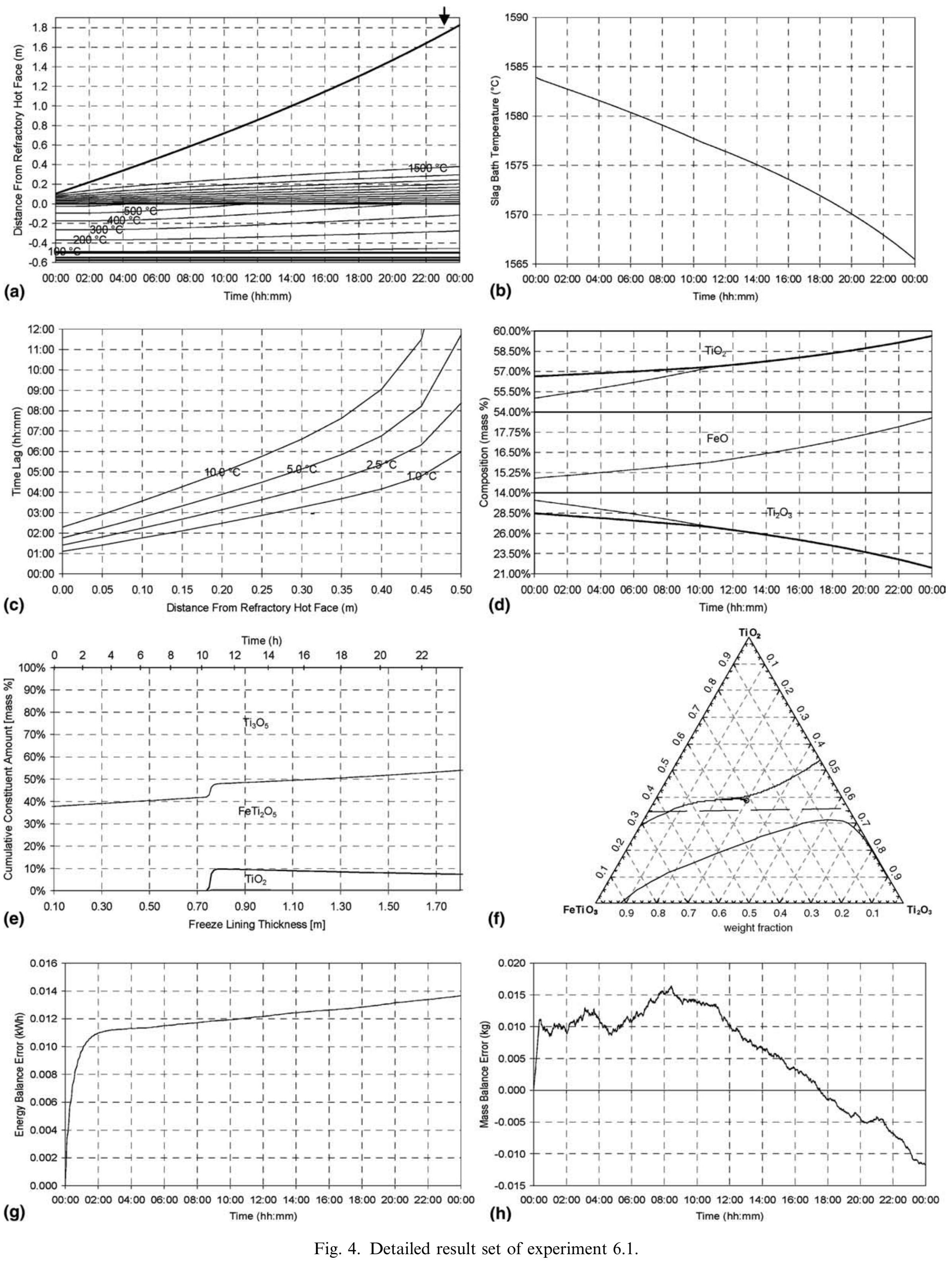


Fig. 4. Detailed result set of experiment 6.1.

4.5. Summary of results

4.5.1. Freeze lining thickness

Fig. 5 shows the influence of net input heat flow rate on freeze lining thickness. An increase in flow rate results in a thinner freeze lining, and a decrease in a thinner freeze lining. When viewing these results, one must take into account that the slag bath was ideally insulated on the top and bottom surfaces and that solidification was only allowed at the freeze lining hot face.

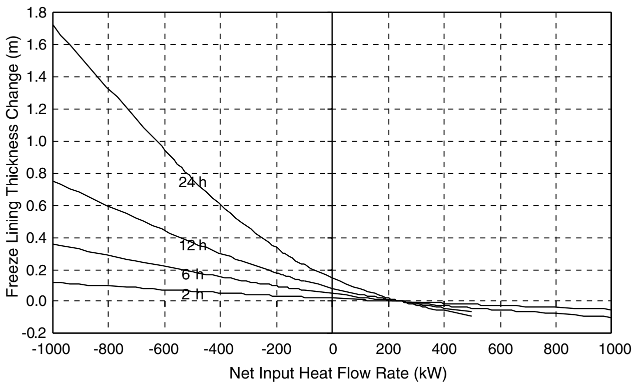


Fig. 5. Change in freeze lining thickness as a function of net input heat flow rate.

The results of the simulation experiments confirmed that too high heat flow rates can melt away the freeze lining in a few minutes, and that it takes hours to rebuild a freeze lining after it had been lost.

4.5.2. Slag bath composition

The behaviour of the slag composition was as expected, given the phase chemistry characteristics of the system. Solidification resulted in an increase in TiO_2 and FeO content, and a decrease in Ti_2O_3 content. Melting of the freeze lining resulted in the opposite, except for those experiments where a pure TiO_2 freeze lining was used as initial condition. In this case only the TiO_2 increased upon melting of the freeze lining, while the content of the other two constituents decreased.

Upon sufficient solidification, the liquid slag composition converged into and followed the eutectic groove present on the ternary phase diagram. This is once again simply a result of the phase chemistry characteristics of the system. This is shown in Fig. 4(d) and (f). In (d) the thick solid TiO_2 and Ti_2O_3 curves indicate the eutectic groove composition, and the thin curves of all three constituents indicate the model results. In (f) the top thin solid curve indicates the eutectic groove, the broken line indicates the line of M_3O_5 composition connecting Ti_3O_5 and $FeTi_2O_5$, and the bottom thin solid curve indicates the edge of the area (below the line) where iron metal is stable. The thick solid curve shows the model results.

4.5.3. Slag bath temperature

The observed slag bath temperature was a result of the phase chemistry characteristics of the system, and the interaction between the freeze lining and slag bath. Because the model used free energy minimisation to continually calculate an equilibrium between part of the slag bath and part of the freeze lining, the slag bath temperature ended up closely representing the liquidus temperature of the slag contained in the slag bath. The downward trend in Fig. 4(b) can therefore be understood when one takes into account that the slag bath composition was moving towards and down the eutectic groove.

4.5.4. Freeze lining composition

When viewing Fig. 4(e) the phase chemistry characteristics of the system are again demonstrated. The thick curve on this graph separates the pseodobrookite at the top from the rutile at the bottom and it indicates the relative amounts of these phases. A thin curve is used to indicate the relative amounts of the constituents within a phase (for example Ti_3O_5 and $FeTi_2O_5$ in pseudobrookite).

The appearance of the rutile phase in Fig. 4(e) coincides with the arrival of the liquid slag composition at the eutectic groove on Fig. 4(f). At that point, the system moves from an area where two phases are stable (liquid slag + pseudobrookite) to an area where three phases are stable (liquid slag + pseudobrookite + rutile).

4.6. Conclusions

From the above it was concluded that, although the actual furnace is not at thermodynamic equilibrium, a good knowledge of the phase chemistry characteristics of the system would provide valuable insight and guidance when considering freeze lining and slag bath behaviour.

5. Influence of slag bath composition on freeze lining and slag bath interactions

5.1. Objectives

The objectives of the second set of experiments with the Wall Model were exactly the same as the first, except that in the second chemical influences were considered, not thermal influences.

5.2. Experimental setup

The flow sheet shown in Fig. 6 was used to conduct the study. The Slag Bath was filled with a slag of specified composition. The Furnace Wall was initialised with a freeze lining of set thickness and composition, and with a specific steady state temperature profile through all the layers of the conductor module. The Metal Bath started off empty. New liquid slag was continuously supplied at the temperature of the Slag Bath via NewSlagFeed. This minimised the thermal influence of the new slag on the system. It also implies that supercooled slag was added to the system in some cases. The total mass of slag in the system was kept constant by removing liquid slag through SlagTap. This was necessary because the Wall Model is one-dimensional. It can only describe variations in the radial dimension. Slag Bath level vari-

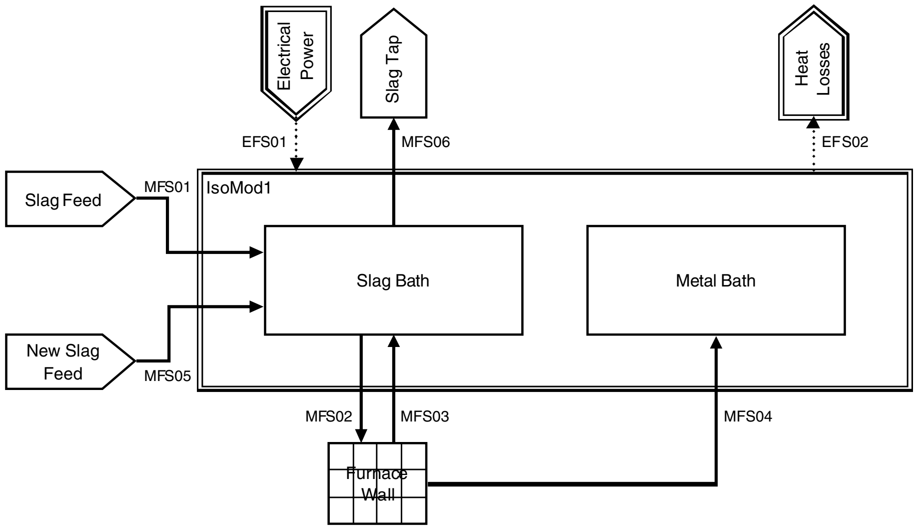


Fig. 6. Flow sheet used to determine influence of slag bath composition on freeze lining and slag bath.

ations therefore had to be eliminated because these result in variations in the axial dimension.

Tables 2 and 3 were again relevant to this set of experiments. All experiments were run for a simulated time of 24 h, or until the freeze lining disappeared.

The following conditions were used in all experiments:

- Initial liquid slag composition: 15%–55%–30% (FeO–TiO_2–Ti_2O_3).
- Liquid slag effective thermal conductivity: 0.005 kW/(m °C).
- Initial steady state heat flow rate through wall: 250 kW.
- Electrical power input: 250 kW.
- Heat loss rate through HeatLosses: 0 kW.
- Initial freeze lining thickness: 0.102 m.
- Initial freeze lining composition: 37.6% $FeTi_2O_5$ and 62.4% Ti_3O_5 (mass basis).

5.3. Experiments conducted

The parameters that were varied over the series of simulation experiments include the following:

- New slag composition.
- New slag feed rate.

The list of experiments is shown in Table 5.

The new slag compositions used are shown in Fig. 7.

5.4. Summary of results

5.4.1. Freeze lining thickness

Because the electrical power input into the system remained constant at 250 kW throughout all experiments, the system's dynamic behaviour would eventually bring it to a steady state of a 250 kW through the furnace wall. Keeping this constant made interpreting the results slightly easier.

Feeding a liquid slag with higher liquidus temperature (compared with the initial liquid slag) firstly caused the slag bath temperature to move towards the liquidus temperature of the incoming slag. Secondly, the freeze lining thickness tended to increase to establish the thermal gradient through the freeze lining and wall that is required to maintain a heat loss rate of 250 kW. The opposite effect was observed when slag with a lower liquidus temperature was fed.

In some cases it appeared that the effect was the opposite of what was just described. Feeding higher liquidus temperature slag resulted in a thinner freeze lining and lower liquidus temperature freeze lining in a thicker freeze lining. The conclusion drawn from such instances was that the effect was only temporary, and that it was the result of the difference in the dynamics of the thermal and chemical phenomena active in the system. While the chemical effect of a composition change became evident virtually immediately due to the slag bath being ideally mixed, the thermal effects were slowed down by conduction heat transfer through the freeze lining and wall. Had the experiments in question been run for longer than 24 h, the expected result would have appeared.

5.4.2. Slag bath composition and temperature

The slag bath composition behaved as one would expect. The composition moved steadily from the initial composition to the composition of the new liquid slag being fed. As with the previous set of experiments, the slag bath temperature followed the liquidus temperature of the slag bath closely, as one would expect.

Table 5
Experiments conducted to study chemical influences with the Wall Model

Experiment no.	New slag composition				New slag feed rate (t/h)	Slag residence time (h)
		FeO	TiO_2	Ti_2O_3		
7.1	A	15.0	50.0	35.0	57.5	4
7.2	B	15.0	60.0	25.0	57.5	4
7.3	C	10.0	55.0	35.0	57.5	4
7.4	D	20.0	55.0	25.0	57.5	4
7.5	E	10.0	60.0	30.0	57.5	4
7.6	F	20.0	50.0	30.0	57.5	4
7.7	G	10.0	50.0	40.0	57.5	4
7.8	H	20.0	60.0	20.0	57.5	4
7.9	I	35.5	59.5	5.0	57.5	4
7.10	J	Pure stoichiometric ilmenite			57.5	4
7.11	A	15.0	50.0	35.0	23.0	10
7.12	B	15.0	60.0	25.0	23.0	10
7.13	C	10.0	55.0	35.0	23.0	10
7.14	D	20.0	55.0	25.0	23.0	10
7.15	E	10.0	60.0	30.0	23.0	10
7.16	F	20.0	50.0	30.0	23.0	10
7.17	G	10.0	50.0	40.0	23.0	10
7.18	H	20.0	60.0	20.0	23.0	10
7.19	I	35.5	59.5	5.0	23.0	10
7.20	J	Pure stoichiometric ilmenite			23.0	10
7.21	A	15.0	50.0	35.0	14.38	16
7.22	B	15.0	60.0	25.0	14.38	16
7.23	C	10.0	55.0	35.0	14.38	16
7.24	D	20.0	55.0	25.0	14.38	16
7.25	E	10.0	60.0	30.0	14.38	16
7.26	F	20.0	50.0	30.0	14.38	16
7.27	G	10.0	50.0	40.0	14.38	16
7.28	H	20.0	60.0	20.0	14.38	16
7.29	I	35.5	59.5	5.0	14.38	16
7.30	J	Pure stoichiometric ilmenite			14.38	16

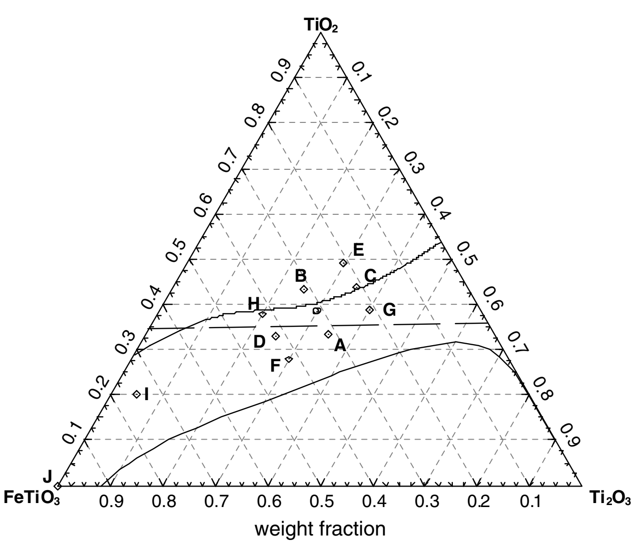


Fig. 7. New slag compositions used to study chemical influences with the Wall Model.

5.4.3. Freeze lining composition

The freeze lining composition was determined by the position of the liquid slag composition on the ternary diagram. When the liquid slag composition was below the eutectic groove, pseudobrookite solidified. When it was on the eutectic groove, both pseudobrookite and rutile solidified. In cases when the liquid slag composition crossed over the eutectic groove, rutile became the only phase that solidified. All of this behaviour can be predicted from the phase chemistry characteristics of the system.

5.5. Conclusions

The important contribution of the phase chemistry characteristics of the system to the behaviour of the system was again evident from this set of experiments. It was also observed that the combination of the thermal and chemical dynamics of the system in some cases results in system behaviour contrary to what is expected. Such contradictions will, however, in time be corrected when the slower thermal dynamics catch up with the chemical dynamic effects.

6. The Crust Model

A model very similar to the Wall Model was constructed to describe the formation of a solid slag crust on top of the slag bath (Zietsman, 2004). The Crust Model only consisted of a layer of solid slag, with no refractory, ramming or steel layer. The Crust Model was one-dimensional in the axial direction and the Wall Model in the radial dimension. Finally, the Crust Model employed an effective convection heat transfer coefficient boundary condition on its cold face, while the Wall Model used a constant temperature boundary condition. The cold face effective heat transfer coefficient was used to quantify the combined effect of convection and radiation. The value of this boundary condition was 0.019 kW/(m^2 °C) when the furnace is operational, and 0.027 kW/(m^2 °C) when the furnace is off. These values were derived from work by Reynolds (2002).

The Crust Model was not used to conduct experiments to study isolated phenomena as the Wall Model was. It was simply constructed for inclusion into the Furnace Model. For this reason no more details on the Crust Model is presented here.

7. The Furnace Model

7.1. Purpose

The purpose of this model was to characterise the dynamic behaviour of and interactions between the freeze lining and slag bath in response to changes in furnace operating parameters.

7.2. Modelled system

The system described by the Furnace Model is shown in Fig. 8. See Tables 2 and 3 for details about the dimensions and material properties used in the Furnace Model.

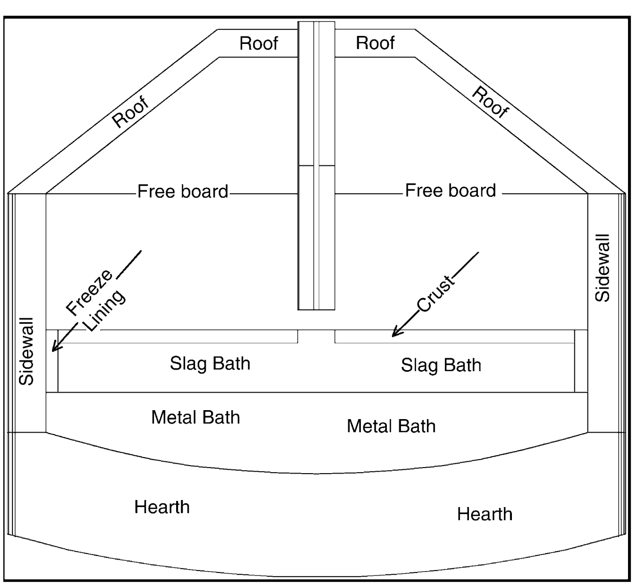


Fig. 8. Schematic of the furnace and process as described by the Furnace Model.

7.3. Key phenomena

In the same way as was done for the Wall Model, phenomena were identified and then prioritised for the Furnace Model. A summary of the results are shown in Table 6.

7.4. Formulation

7.4.1. Assumptions

The following assumptions were made as part of the formulation of the Furnace Model:

- The slag bath, metal bath and furnace freeboard are all at the same temperature.This is definitely not fully representative of reality. In reality there should be a temperature difference between the slag bath surface and the freeboard, and between the slag bath and the metal bath. However, this assumption was made to simplify the system that was modelled here. These simplifications will be eliminated in future models.
- The slag bath, metal bath and freeboard are ideally mixed.
- Gas produced by any form of reduction reaction in the process immediately escapes into the freeboard and does not come into contact with slag, metal or reductant again.
- No air enters the furnace during operation and periods of down time.

7.4.2. Simplifications

The following simplifications were made in the formulation of the Furnace Model:

- The model was configured in such a way that slag and metal were tapped continuously from the furnace. This was a result of the restrictions of the one-dimensional Wall Model. This model is not compatible with axial variations resulting from changing slag and metal bath levels.
- A collection of stoichiometric condensed phases was used to describe ilmenite instead of a collection of solid solution phases.

7.4.3. Model structure

The Furnace Model was constructed according the modelling framework proposed by Pauw (1989). The flow sheet is shown in Fig. 9.

Details of this flowsheet are published elsewhere (Zietsman, 2004). Only the most important aspects are highlighted here.

Table 6
Summary of key phenomena for the Furnace Model

			Level of Importance		
			1	2	3
Heat transfer	From the arc			✓	
	Between slag bath and metal bath		✓		
	From slag bath surface to freeboard			✓	
	From slag bath to freeze lining and sidewall				✓
	From slag bath to crust and freeboard				✓
	From metal bath to sidewalls and hearth			✓	
	Through upper sidewalls and roof			✓	
Mass transfer	Bulk flow	Feed materials			✓
		Product materials			✓
		Metal droplets through slag bath	✓		
		Gas from slag bath surface into freeboard	✓		
		Gas through slag bath into freeboard	✓		
		Process material to sidewalls and roof	✓		
		Process material from sidewalls and roof into slag bath	✓		
	At slag–reductant interface			✓	
	At interface between slag and metal baths			✓	
	Carbon into metal bath			✓	
	At interface between slag bath and freeze lining			✓	
Momentum transfer	Momentum sources	The arc	✓		
		Feed material	✓		
		Electromagnetic stirring	✓		
		Rising gas	✓		
		Buoyancy forces	✓		
	In the slag bath			✓	
	In the metal bath			✓	
	In the freeboard		✓		
Chemical reaction	Liquid slag	With ilmenite			✓
		With reductant			✓
		With liquid slag			✓
		With liquid metal			✓
		With refractory material	✓		
	Solid slag	With liquid slag			✓
		With solid slag	✓		
		With liquid metal	✓		
	Liquid metal	With reductant		✓	
		With refractory material	✓		
	Reduction product gas	With liquid slag	✓		
		With reductant	✓		
	Air		✓		
	Graphite		✓		
Mechanical effects	Slag and metal tapping		✓		
	Cave-in of accretions		✓		
	Water leaks		✓		

HeatLoss1 represents a portion of the heat loss from the process that is lost through the upper sidewalls and roof. HeatLoss2 represents heat losses from the process through the lower sidewalls and hearth. The Crust Model extracts heat from the system at a rate which depends on the slag bath surface area and temperature, through the effective heat transfer coefficient boundary condition as mentioned earlier. The Wall Model also extracts heat from the system at a rate which depends on the boundary conditions, freeze lining thick-

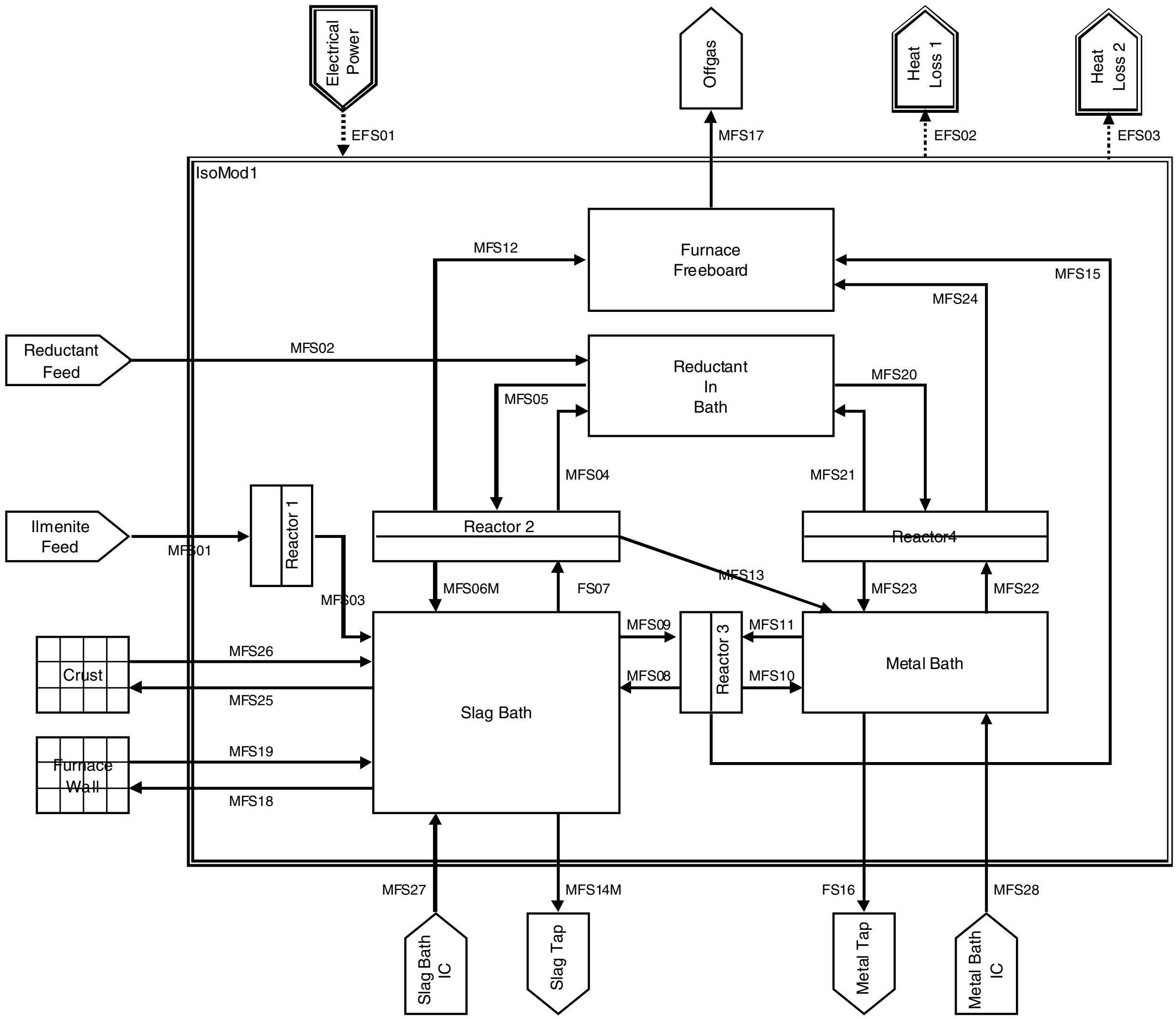


Fig. 9. Flowsheet of the Furnace Model.

ness, and transient effects. The combination of these four modules covers the entire heat loss boundary around the system, to quantify the heat loss from the total system.

IsoMod1 represents virtually the entire content of the furnace. The only exclusions from it are the solid slag of the freeze lining and the solid slag of the crust when a crust is present. Since these parts of the process contain temperature gradients that are important to the current study, they could not be described using an isothermal module.

The SlagBathIC and MetalBathIC material input modules do not refer to any physical part of the actual process. They were included into the model to generate an initial condition (IC) of the slag bath and metal bath states.

Reactor1 represents the melting of ilmenite occurring in the process as this feed material heats up. Reactor2 represents all reduction reactions taking place at an interface between liquid slag and reductant. Reactor3 represents all reduction reactions taking place at an interface between liquid slag and liquid metal. Reactor4 represents contact between reductant particles and liquid metal in the turbulent zone underneath the electrode. The contact results in dissolution of carbon in liquid metal.

The flowsheet shown in Fig. 9 represents a set of differential equations, which was solved using Euler integration.

7.5. Validation

Due to its complexity, the Furnace Model could not be validated against a known analytical solution. Due to the confidentiality of all ilmenite-smelting operations, the model could also not be validated against industrial data. The only option left was to use the authors' experience on pilot-scale and industrial scale ilmenite furnaces as the basis of validation. Numerous validation experiments were done and adjustment to model parameters made. The end result was a model that fairly represents an industrial-scale smelting furnace. The final model parameters were documented in the complete study report (Zietsman, 2004).

8. Influence of operating parameters

8.1. Objectives

In the simulation experiments conducted with the Furnace Model, it was aimed to study the influence of step changes in operating parameters on the dynamic behaviour of the freeze lining, furnace wall, crust and slag bath. The following aspects were investigated and are reported on here:

- Freeze lining and crust thickness.
- Temperature distribution through the furnace wall, freeze lining and crust.
- Liquid slag temperature.
- Composition distribution through the freeze lining.
- Liquid slag composition.

8.2. Experimental setup

The flow sheet shown in Fig. 9 was used to conduct the study. The following steady state was calculated, and all of the experiments used it as starting condition:

- INPUTS
  - ElectricalPower: 21,715 kW
  - IlmeniteFeed: 20,000 kg/h
  - ReductantFeed: 1700 kg/h
- CONTENTS
  - IsoMod1:
    - Temperature: 1589 °C
  - SlagBath:
    - Mass: 225,859 kg
    - Composition: 14.7–54.0–31.9 (FeO–TiO_2–Ti_2O_3 mass percentages)
  - MetalBath:
    - Mass: 323,002 kg
    - Composition: 2 mass percent carbon. Balance as Fe.
  - ReductantInBath:
    - Mass: 52.4 kg
  - FurnaceFreeboard:
    - Mass: 30.9 kg
    - Composition: 98.8–1.2 (CO–CO_2 volume percentages)
  - FurnaceWall:
    - Freeze lining thickness: 76.3 mm
    - Freeze lining mass: 7992 kg
  - Crust:
    - Crust thickness: 0 mm
- OUTPUTS
  - HeatLoss1: 161.5 kW
  - HeatLoss2: 434.8 kW
  - FurnaceWall: 280 kW
  - Crust: 2631 kW
  - SlagTap: 11,842 kg/h
  - MetalTap: 6151 kg/h
  - Offgas: 3707 kg/h

The effective thermal conductivity of the liquid slag was assumed to be 0.005 kW/(m °C).

All experiments were run for a period of 24 h, or until the freeze lining had been melted away completely.

8.3. Experiments conducted

The experiments conducted are listed in Table 7. It consists of three subsets. The first subset (8.1 and 8.2) tested the influence of severe operational errors. The second subset (8.3–8.10) tested the influence of independent adjustment of energy and reductant inputs. The third subset (8.11–8.20) tested the influence of appropriately combined adjustment of energy and reductant inputs.

Detailed results of each experiment have been published (Zietsman, 2004). Only the summarised results of the three subsets are presented below.

9. Influence of operating parameters—severe operational errors

In experiment 8.1 all material inputs were stopped while the electrical energy input was maintained. In experiment 8.2 only the reductant input was stopped and the ilmenite and electrical energy inputs were maintained. These scenarios can occur in the event of partial of complete feed system failure.

9.1. Summary of results

9.1.1. Freeze lining thickness

The freeze lining disappeared completely 19 min after stopping all feed in experiment 8.1. It took 36.5 min to melt away the freeze lining after stopping only the reductant feed. The reason for the delay in the second experiment was the fact that ilmenite was still being added and consuming energy for heating and melting at around 11,000 kW.

If an operational error such as those simulated here would go unnoticed for periods longer than what is mentioned above, serious damage can be done to the sidewall refractory material. The onset of such damage

Table 7
Experiments conducted to study the influences of operation parameters with the Furnace Model

Experiment no.	Ilmenite feed rate change (kg/h)	Ilmenite feed rate (kg/h)	Reductant feed rate change	Reductant feed rate (kg/h)	Electrical power change (kW h/ton ilm.)	Electrical power (kW)
8.1	−20,000	0	−1700 kg/h	0	–	21,715
8.2	–	20,000	−1700 kg/h	0	–	21,715
8.3	–	20,000	+1 kg/ton ilm.	1720	–	21,715
8.4	–	20,000	−1 kg/ton ilm.	1680	–	21,715
8.5	–	20,000	+2 kg/ton ilm.	1740	–	21,715
8.6	–	20,000	−2 kg/ton ilm.	1660	–	21,715
8.7	–	20,000	–	1700	+5	21,815
8.8	–	20,000	–	1700	−5	21,615
8.9	–	20,000	–	1700	+10	21,915
8.10	–	20,000	–	1700	−10	21,515
8.11	–	20,000	+1 kg/ton ilm.	1720	+5	21,815
8.12	–	20,000	−1 kg/ton ilm.	1680	−5	21,615
8.13	–	20,000	+2 kg/ton ilm.	1740	+10	21,915
8.14	–	20,000	−2 kg/ton ilm.	1660	−10	21,515
8.15	–	20,000	+5 kg/ton ilm.	1800	+25	22,215
8.16	–	20,000	−5 kg/ton ilm.	1600	−25	21,215
8.17	–	20,000	+10 kg/ton ilm.	1900	+50	22,715
8.18	–	20,000	−10 kg/ton ilm.	1500	−50	20,715
8.19	–	20,000	+10 kg/ton ilm.	1900	+52	22,755
8.20	–	20,000	−10 kg/ton ilm.	1500	−53	20,655

can even be sooner than what is mentioned above because of an actual freeze lining not having a constant thickness around the circumference of the furnace. Damage would most likely start sooner in the vicinity of the tapholes.

9.1.2. Slag bath temperature

The slag bath temperature showed a significant increase during both experiments (8.1 and 8.2). Upon closer investigation it was found that the slag bath temperature did not follow the liquidus temperature of the slag bath as it did during previous experiments with the Wall Model. Due to the inability to validate the Furnace Model against industrial data, it is uncertain whether this will happen in reality.

If the model results are qualitatively correct in predicting that the slag bath would be superheated, the impact of the operational errors simulated here would be even more severe than the loss of freeze lining and some refractory material. These errors appear to build up 'momentum' in the form of superheat. This superheat needs to be dissipated well after the furnace power had been switched off. During this time much more refractory material could lost into the slag bath.

9.1.3. Slag bath composition

The compositional change of the slag bath in the first experiment was mostly the result of solid slag of the freeze lining melting into the slag bath. Due to the relative masses of the initial freeze lining and the liquid bath, the change was slight.

During the second experiment (8.2) the slag bath composition changed due to solid slag melting into the slag bath, and ilmenite entering into the slag bath. The influence of the ilmenite dominated the change and it drew the slag bath composition toward the $FeTiO_3$ corner of the ternary diagram.

9.2. Conclusions

Severe operational errors such as those described above (losing all feed, only all reductant or only all ilmenite) should be avoided if at all possible. If such an error is detected, it is imperative to act as quickly as possible to counter its effect.

If one is able to detect the error early, it may be sufficient to stop all process inputs, most importantly electrical power. The extra energy discharged into the process will then be dissipated to the freeboard, metal bath and freeze lining. If it was detected in time, the entire freeze lining would not melt away.

If the error was not detected early, the slag bath could have been superheated significantly. Only switching off the furnace entirely may not be enough to protect it. The superheated slag will continue to dissolve the solid slag from the freeze lining and even an appreciable mass of refractory material from the furnace sidewalls.

In such an event one must attempt to re-establish or maintain the two largest process heat sinks. The largest is the heating and melting of ilmenite (approximately 11,000 kW for the setup used here). If ilmenite feed failed, it should be rectified as soon as possible and restarted. From the modelling work, the second-largest heat sink appears to be the reduction reactions at the reductant–slag interface (approximately 5500 kW). If the reductant feed failed, it should be restarted as quickly as possible.

The question comes to mind as to whether one should completely stop the electrical power input or not. If one switches on ilmenite and reductant feed at full rates without power, it may turn out to be somewhat ineffective. The reason is that the material may not enter into the slag bath as well as it would when the force of the arc is present to drive it into the slag bath. Ilmenite and reductant may float on the surface of the slag bath as islands of agglomerated materials. For this reason it may make sense to keep electrical power switched on at a low level in an attempt to maximise the heat sinks. Only practical experience will be able to tell which combination would be most effective.

10. Influence of operating parameters—independent adjustment

It was reported by Pistorius (1999) that energy and reductant inputs cannot be changed independently while maintaining stable process conditions. Pistorius (1999) further proposed that a ratio of 1 kg of reductant to 5 kW h of energy be used when making setpoint changes if one wants to keep the process in balance.

This ratio was ignored in the second subset of experiments conducted with the Furnace Model in an attempt to test the validity of the suggested ratio.

10.1. Summary of results

10.1.1. Freeze lining thickness

The independent changes in operating parameters resulted in changes in freeze lining thickness. A decrease in electrical power and an increase in reductant feed rate caused the freeze lining to become thicker. Changes in the opposite direction had the opposite effect.

10.1.2. Crust thickness

In cases when the effective energy input was substantially reduced (by reducing electrical power or by increasing reductant feed rate), the model indicated the formation of a crust. This may in fact happen in reality, or it may be indicative of the onset of slag foaming. Solid slag particles may precipitate in the slag bath, increase the apparent viscosity of the liquid slag and result in foaming.

The fact that the model was able to indicate crust formation as a result of such relatively subtle operational changes was unexpected. The prediction of solids formation and the onset of foaming do however correlate well with practical observations under comparable operational conditions. This gives additional confidence in the model and its qualitative validity.

10.1.3. Slag bath composition

An independent increase in reductant feed rate caused an increase in Ti_2O_3 and a decrease in FeO and TiO_2. An independent decrease resulted in the opposite effect. Quantitative knowledge of these influences can be used when it is required to rectify an imbalance in the current operational conditions. This type of knowledge is what is required when one wants to automate control of the furnace by implementing an advanced process control system.

10.2. Conclusions

Independent adjustment of operational parameters moves the process away from the current stable point of operation. In some cases it could result in process instabilities such as foaming and crust formation. In other cases it could result in loss of freeze lining and associated damage to sidewall refractory material.

Intelligent use of the knowledge of the effects of such adjustments could valuable in situations when one wants to rectify operational imbalances. Further work with the Furnace Model can be used to characterise these adjustments accurately enough to apply them automatically through an advanced process controller.

11. Influence of operating parameters—balanced adjustment

The 1:5 ratio proposed by Pistorius (1999) was tested further by implementing and refining it in the third subset of Furnace Model experiments.

11.1. Summary of results

11.1.1. Freeze lining thickness

If the 1:5 ratio proposed by Pistorius (1999) was in fact accurate, the freeze lining thickness would have remained constant during the experiments in the third subset. This was not the case. There were slight changes in thickness. The last two experiments were used to fine-tune the ratio. The result of this tuning was a ratio of between 1:5.2 and 1:5.3, which is very close to what Pistorius proposed. These values reduced the changes in freeze lining thickness substantially.

Because a simplified three-component system was used in the modelling work described here, the ratios mentioned above will have to be recalculated for real operations. Knowledge of these ratios could prove very valuable when aiming to achieve stable furnace control.

11.1.2. Slag bath temperature and composition

Slag bath temperature and Ti_2O_3 content increased and FeO and TiO_2 content decreased when reductant and energy input was increased according to the 1:5 ratio. A reduction in reductant and energy inputs according to the ratio had the opposite effect. These results were expected, and supports the notion that one can use the 1:5 ratio to move the furnace from one point of operation (for example high FeO) to another (lower FeO) in a stable and controlled manner.

11.2. Conclusions

The 1:5 ratio proposed by Pistorius (1999) appears to be remarkable accurate when applied to the simplified system used here. It also appears that this ratio, after accurate determination for a real operation, can be applied to improve control of the process and even to automate control of the slag bath composition. This could very likely result in significant improvements in process performance.

12. Conclusions

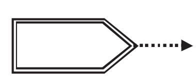
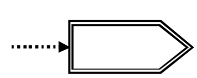
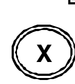
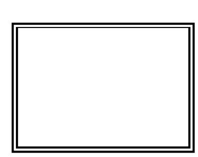
This paper presented modelling work on an ilmenite-smelting DC arc furnace. The main focus of the study was the interactions between the freeze lining and slag bath. This was studied by determining the influences of thermal changes, chemical changes and changes in operational parameters.

The work has some limitations in that the model used to describe the freeze lining and furnace wall is one-dimensional in the radial dimension. Variations in metal and slag bath levels could therefore not be described. Further, the models could not be validated against industrial data because of the confidentiality of ilmenite-smelting operations. The experience of the authors with pilot- and industrial-scale ilmenite-smelting furnaces was used to counter this restriction.

The modelling results yielded numerous useful insights into the behaviour of the freeze lining, slag bath and the process as a whole. These insights can be used directly in the improvement or refinement of operational strategies. It can also be developed further and integrated into an advanced process control system that could perform process calculations, such as those done by the models presented here, on-line and implement setpoint changes in closed loop to improve aspects of process performance such as throughput, variation in slag composition, recovery from periods of down time, etc.

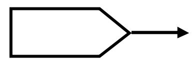
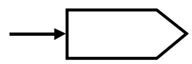
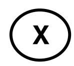
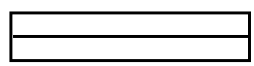
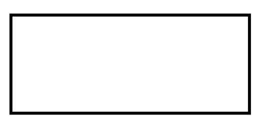
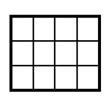
Before the models could be applied to real industrial processes, they would have to be validated and tuned to quantitatively represent those processes with a fair degree of accuracy.

Appendix A. Model element descriptions

Name	Abbr.	Symbol	Descreption
Energy Flow Stream	EFS		An energy flow stream is used to connect modules with energy output ports and energy input ports. The flow stream receives energy from an output port and passes it to an input port
Energy Input Module	EIM		An energy input module produces energy. The produced energy is made available at its output port where an energy flow stream receives it
Energy Output Module	EOM		An energy output module extracts energy. It receives energy at its input port from an energy flow stream
Energy Fraction Splitter	EXS		An energy fraction splitter is used to split the energy from one energy flow stream into two or more fractions. It makes the fractions available at its output ports where it is received by two or more energy flow streams
Isothermal Module	ITM		An isothermal module represents a zone in a process that can be approximated as being isothermal. This module contains MMM, MRM, MXS and MPS modules. All the contained MMM modules are given the same temperature
Material Flow Stream	MFS		A material flow stream is used to connect modules with material output ports and material input ports. The flow stream receives material from an output port and passes it to an input port

(*continued on next page*)

Appendix A (*continued*)

Name	Abbr.	Symbol	Description
Material Input Module	MIM		A material input module produces material. The produced material is made available at its output port where a material flow stream receives it
Material Output Module	MOM		A material output module extracts material. It receives material at its input port from a material flow stream
Material Fraction Splitter	MXS		A material fraction splitter is used to split the material from one material flow stream into two or more fractions. It makes the fractions available at its output ports where it is received by two or more material flow streams
Material Phase Splitter	MPS		A material fraction splitter is used to split multiphase material into single-phase material. The number of output ports is equal to the number of phases in the material received at its input port
Material Reactor Module	MRM		A material reactor module is used to calculate the equilibrium condition of the material received at is input ports. The resulting material is delivered at its output ports as single-phase material
Material Mixer Module (Ideal Mixer)	MMM		A material mixer module is used to represent a collection of material in a process that can be approximated as being ideally mixed. It can receive material at multiple input ports and deliver material to multiple output ports
Conductor Module	CDM		A conductor module is used to conduction calculate heat transfer through, for example, a wall. It exchanges liquid material with a mixer module. It models solidification and melting of this material

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